

# Complex Systems

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Khordad 1401

Examination for the complex systems course. Answer all three questions. Total time allowed is 2 hours. The number in parenthesis is the mark for that problem.

1. (5) If the differential equation  $\frac{dy}{dx} = \alpha y x^\beta$ ,  $\alpha, \beta \in \mathbb{R}$ ; admits scaling solutions, determine the scaling exponent of  $y(x)$ . What happens when  $\beta = 1$
2. (5) Show that if the volume of an ideal gas composed of  $N$  particles is scaled by a factor  $\lambda$ , the increase in entropy is  $\Delta S = N \log \lambda$ .
3. (7) The complete tripartite graph  $K_{rst}$  consists of three sets of vertices (of sizes  $r$ ,  $s$ , and  $t$ ), with an edge joining two vertices if and only if they lie in different sizes. Draw the graphs  $K_{222}$  and  $K_{332}$ . Then find the number of edges of  $K_{345}$ . Find a general expression for the number of edges.